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Comparison of deflection calculations and span-to-depth ratios in BS 8110 and Eurocode 2

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The current paper proposes a procedure for calculating deflections with Eurocode 2, which is subsequently used to evaluate the span-to-effective-depth rules in Eurocode 2 and British Standard (BS) 8110. The theoretical background to span-to-depth rules is examined. The Eurocode 2 span-to-depth rules were derived by curve-fitting the results of a parametric study in which deflections were calculated in simply supported beams under a permanent load equal to half the design ultimate load. The loading assumed in the derivation of the Eurocode 2 span-to-depth rules is unrealistic for lightly reinforced slabs where self-weight can exceed half the design ultimate load. The procedure used to modify the Eurocode 2 span-to-depth rules for different stress levels is also shown to be questionable. An amendment is proposed that is shown to improve the accuracy of the Eurocode 2 span-to-depth rules.

Notation

A_{sprov}	area of reinforcement provided at mid-span
A_{sreq}	area of reinforcement required for strength at mid-span
$f_{ckequiv}$	equivalent concrete compressive strength to be used in span-to-depth rules
f_{cm}	mean concrete compressive strength
h	slab thickness: mm
K	relates the permissible span-to-depth ratio to structural form
M	maximum span moment
M_{ext}	moment at the external support
M_r	cracking moment
M_{rpeak}	cracking moment when peak construction load is applied
t	time
w_{con}	construction load
w_d	dead load
w_{freq}	frequent load
w_i	imposed load
w_{perm}	permanent load
w_u	design ultimate load
X	relates deflection to the loading arrangement and boundary conditions
γ_s	$= 1{\cdot}15$ where γ is the partial factor for reinforcement
ρ	required tension steel ratio at mid-span
ρ_0	reference reinforcement index $\sqrt{f_{ck}} \times 10^{-3}$
ρ'	required compression steel ratio at mid-span
σ_s	reinforcement stress
Ψ_m	mean curvature
Ψ_1, Ψ_2	curvatures in idealised uncracked and fully cracked sections respectively

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Calculation of deflection in Eurocode 2

Eurocode 2[1] calculates the mean curvature in cracked reinforced concrete members by interpolating between the curvatures in idealised uncracked and fully cracked sections as follows

$$\Psi_m = \zeta\Psi_2 + (1–\zeta)\Psi_1 \tag{1}$$

where

$$\zeta = 1–\beta(M_r/M)^2 \tag{2}$$

Ψ_1 and Ψ_2 are the curvatures in idealised uncracked and fully cracked sections respectively including shrinkage and M_r is the cracking moment when the moment M is first applied. The coefficient β in Equation 2 accounts for the loss of tension stiffening with time owing to additional internal- and macro-cracking under sustained load. Eurocode 2[1] states that β should be taken as 1 for short-term loading and 0·5 for long-term loading but does not define the variation in β with time.

Back-analysis of deflection data from laboratory tests and field measurements[2–6] shows that Equation 1 gives good estimates of curvature and hence deflection if the material properties and loading are known. Difficulties arise in practice, as neither the concrete material properties nor the actual loading are known prior to construction or indeed subsequently. Deflections in slabs are particularly difficult to estimate reliably because they vary significantly dependent on whether or not the slab is cracked. Research[2–6] suggests that deflections in slabs are largely governed by the most severe cracking, which can arise during construction or subsequently in service. Cracking can arise during construction either at striking, or subsequently under loading from casting slabs above or stockpiling construction materials.

Construction loading from casting slabs above

In multi-storey buildings, the self-weight of the most recently cast slab is transferred into the slabs below through props and backprops until the most recently cast slab carries its self-weight. The magnitude of the load induced in slabs from casting slabs above depends on the type of formwork system, the number of levels of backprops and their spatial arrangement. Peak construction loads from casting slabs above depend on whether slabs carry their self-weight before the slab above is cast. Beeby[7] showed through analysis and measurements of prop forces at Cardington that the peak construction load occurs in the top slab of the supporting assembly when slabs carry their self-weight before the slab above is cast. The lower slab in the supporting assembly is most heavily loaded if slabs are not struck (i.e. carry their own weight) before the slab above is cast. If the most recently cast slab carries its self-weight after striking, the peak construction load is given by

$$w_{\text{peak}} = w_{\text{self}} + c(w_{\text{self}} + w_{\text{con}}) \tag{3}$$

where c is a carry-through factor of at least 1/(number of supporting floors) and w_{con} is a construction load comprising formwork and so on, which is typically around 0·75 kN/m^2. Beeby's[7] work showed that, when backprops are installed fingertight, it is reasonable in the absence of detailed calculation to take c as 0·7 in Equation 3 if there is one level of backprops and 0·65 with two levels of backprops. In practice, backprops are usually preloaded during installation rather than being installed fingertight as at Cardington. The current author measured significant preloads in the backprops at St George Wharf,[5] which corresponded to a uniformly distributed load of ~1 kN/m^2. Preload is beneficial because it induces a more even distribution of construction load between the supporting slabs than measured at Cardington. Parametric studies indicate that it is reasonable to take the peak construction load w_{peak} as 0·04h kN/m^2 (where h is the slab thickness in mm) in deflection calculations for slabs up to 500 mm thick where two levels of backprops are used and the backprops are preloaded during installation as at St George Wharf.[5]

Construction loads from casting slabs above can only be neglected if (*a*) the formwork is supported by the columns or (*b*) sufficient backprops are provided to transfer the self-weight of the most recently cast slab into the ground. Caution should be exercised in neglecting construction loads because measurements of prop forces at Cardington[3] and St George Wharf[5] suggest that slabs can experience significant construction loads from casting slabs above even if the backprops continue to the ground owing to the combined effects of prop shortening and ground settlement.

Pallet[8] gives detailed guidance on designing slabs for peak construction loads at the ultimate limit state. He suggests that the load factor can be taken as 1·2 for dead and imposed loads during construction and that reduced material factors of safety can be used at the ultimate limit state if the worst credible rather than characteristic material strengths are used. Shear failure is likely to be most critical during construction because the flexural strength in lightly reinforced members is almost independent of the concrete compressive strength.

Influence of peak loads on long-term deflection

The current author (see Vollum and Afshar[6]) recently tested a series of six simply supported one-way spanning slabs to determine the influence of short-term construction loads on long-term slab deflections. The slabs measured 500 mm wide by 3600 mm long by 150 mm thick and were reinforced with 3T10 bars with 20 mm cover. The slabs were simply supported over a span of 3300 mm and loaded at their third points. Full details of the tests are given elsewhere.[6] The mid-span moment and curvature are plotted against time in Figures 1 and 2 respectively for slabs S4 to S6. The peak construction moment M_{peak} varied in tests S4 and S5. Slab S6 was a control specimen, which was not loaded with a peak construction load. The sustained load was the same in tests S4 to S6. Figure 2 shows that long-term curvatures in reinforced concrete slabs can be

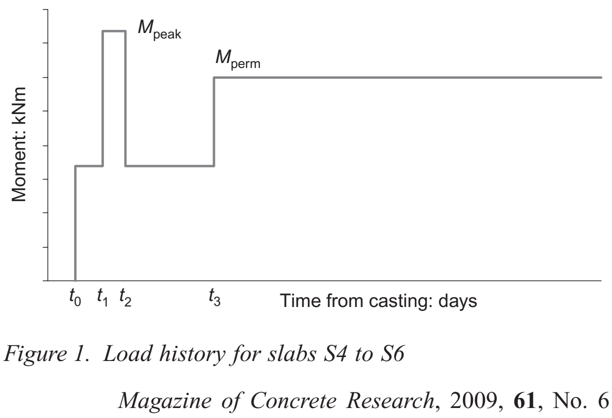


Figure 1. Load history for slabs S4 to S6

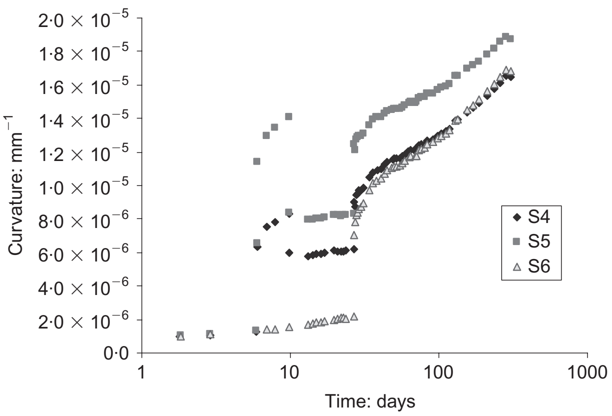


Figure 2. Comparison of deflections in slabs S4 to S6

significantly influenced by previous loading which is uncertain in practice.

Rigorous method for deflection prediction

Eurocode 2[1] states that deflections should be calculated under the quasi-permanent load, which is exceeded during half the design period, and is given by

$$w_{\text{perm}} = w_{\text{d}} + \psi_2 w_{\text{i}} \tag{4}$$

where w_{d} is the dead load and w_{i} is the imposed load, ψ_2 is taken as 0·3 for offices but depends on usage.

Eurocode 2[1] gives no guidance on how to account for the increase in deflection owing to construction loading illustrated in Figure 2. The current author recently proposed[6] a simple method for taking account of construction loading in deflection calculations, which is a refinement of his earlier work which formed the basis of the rigorous method in Concrete Society Technical Report TR58.[9] The method was validated[6] with deflection data from slabs S1 to S6 described above. It is proposed that mean curvatures are calculated with Equation 1 in conjunction with Equation 2 using equivalent values for the concrete effective elastic modulus and the interpolation coefficient ζ to account for the effects of previous loading. The equivalent elastic modulus of the concrete can be calculated for the concrete using the following equation from TR58[9]

$$E_{\text{LT}} = \Sigma\Delta w_{\text{i}}/(\Delta w_1/E_{\text{ceff1}} + \Delta w_2/E_{\text{ceff2}} + \Delta w_3/E_{\text{ceff3}} + \ldots) \tag{5}$$

where Δw_{i} is the load increment at time t_{i} and

$$E_{\text{ceff}} = E_{\text{ct}}/[1 + \phi(t_{\text{i}}, t)]$$

where t_{i} is the age at application of w_{i} and t is the age at which deflections are required.

The interpolation coefficient ζ used in Equation 1 needs to be modified to take into account the loss of tension stiffening induced by short-term peak loads greater than w_{perm}. Tension stiffening is reduced following the removal of short-term peak loads because (*a*) additional internal and macro-cracks form under w_{peak} and (*b*) the slope of the unloading line is steeper than the instantaneous loading line. The current author[6] has previously shown that the influence of peak construction loads can be included in deflection calculations by replacing ζ in Equation 1 with a modified interpolation coefficient ζ given by

$$\zeta^* = 0{\cdot}5\zeta_{\text{peak}}(1 + M_{\text{peak}}/M) \tag{6}$$

where M is calculated with the load w under which deflections are required and

$$\zeta_{\text{peak}} = 1 - \beta(M_{\text{rpeak}}/M_{\text{peak}})^2 \tag{7}$$

where M_{rpeak} is the cracking moment when the peak construction load is applied and M_{peak} is calculated under the peak construction load. The ratio M_{peak}/M can be taken as w_{peak}/w for uniformly loaded slabs. Back-analysis of deflection data from the author's slab tests,[6] Cardington[2–4] and St George Wharf[5] suggests it will usually be conservative to take β as 0·7 in Equation 7 at the removal of the peak construction load if curvatures are calculated with ζ^*. The term $0{\cdot}5(1 + M_{\text{peak}}/M)$ in Equation 6 for ζ^* accounts for the increment in deflection owing to inelastic unloading from M_{peak}.

The maximum in service load, which is uncertain, may give rise to more severe cracking than the peak construction load if (*a*) the slab is heavily loaded or (*b*) the backprops are continued down to the ground as is typical in low-rise buildings. Theoretically, Eurocode 0 could be interpreted as requiring ζ to be calculated under the full service load. In practice, this may be rather onerous and PD 6687[10] suggests that ζ can generally be calculated under the frequent load combination, which is exceeded during 1% of the design period and is defined as

$$w_{\text{freq}} = w_{\text{d}} + \psi_1 w_{\text{i}} \tag{8}$$

where ψ_1 is 0·5 for offices.

In practice, designers are normally interested in calculating (*a*) maximum deflections and (*b*) incremental deflections following fitting out. Eurocode 2[1] differs from BS 8110[11] in that it requires deflections to be calculated under the quasi-permanent rather than peak load. The change in loading proposed in Eurocode 2[1] seems questionable since damage to finishes and partitions is likely to be governed by the peak increment in deflection subsequent to their installation. Furthermore, it is difficult if not impossible realistically to predict deflections in building structures under quasi-permanent loads because the deflections depend on the previous peak loading, its duration and the ratio of w_{perm} to w_{peak}, all of which are uncertain. It is therefore proposed that the maximum deflection should be calculated under the frequent rather than quasi-permanent load.

Recommended procedure for calculating deflections with Eurocode 2

Difficulties arise in practice, because the loading and concrete material properties can only be estimated at the design stage. It is suggested that in the absence of better information, the following assumptions are made in deflection calculations.

(*a*) The slab is struck at 7 days, the superimposed dead load is applied at 60 days and the permanent component of the imposed load at one year.
(*b*) Creep and shrinkage strains are calculated with a relative humidity of 50%.
(*c*) Two levels of backprops are used.
(*d*) The floor above is cast after 10 days.
(*e*) When slabs are supported by slabs below during construction, the peak construction load w_{peak} should be taken as $0{\cdot}04h$ kN/m^2 where h is the slab thickness in mm.
(*f*) The permanent load should be taken as the quasi permanent load and be applied at 1 year.
(*g*) Peak deflections are calculated under the frequent load case; the increment in load $w_{\text{freq}}-w_{\text{perm}}$ should be treated as an instantaneous load in Equation 5 for E_{LT}.

Choice of concrete tensile strength

It is difficult to assess the effective tensile strength of concrete in slabs owing to its inherent variability and uncertainties in the tensile stress induced by internal and external restraint of shrinkage. Back-analysis of deflection data shows that the effective flexural strength of concrete in reinforced concrete slabs typically lies somewhere between the indirect and flexural strengths. Analysis of test data[2,5,6] shows that the Eurocode 2 formula for the mean concrete tensile strength is rather conservative as it appears to calculate the tensile strength in terms of the characteristic rather than mean concrete compressive strength. Back-analysis of deflection data from Cardington[2–4] and St George Wharf[5] suggests that it is reasonable to calculate the concrete tensile strength in deflection calculations in terms of the mean concrete compressive strength as follows

$$f_{\text{ctm28eff}} = 0{\cdot}3 f_{\text{cm}}^{2/3} \tag{9}$$

where f_{cm} is the mean concrete compressive strength at time $t \leqslant 28$ days. Eurocode 2 defines the mean compressive strength at 28 days as $f_{\text{ck}} + 8$. An alternative approach[7] is to take the concrete tensile strength as the mean of the tensile and flexural strengths calculated as follows in terms of the characteristic cylinder strength

$$f_{\text{ctm28eff}} = 0{\cdot}3 f_{\text{ck}}^{2/3}(1{\cdot}3 - h/2000) \tag{10}$$

Parametric studies show that Equations 9 and 10 give similar strengths for slabs up to 400 mm thick. Equation 9 is arguably preferable since the proportional reduction in effective tensile strength owing to axial restraint from columns and shear walls is likely to be greatest in thin slabs.

Calculation of incremental deflection following fitout

The incremental deflection experienced by the partitions and other finishes depends significantly on whether slabs crack before or subsequent to fitting out. Cracking during construction tends to increase the overall deflection but frequently reduces the subsequent increase in deflection as illustrated in Figure 2. The increment in deflection Δa seen by partitions and other finishes can be conservatively calculated as follows

$$\Delta a = a_\infty - a_1 \tag{11}$$

where a_∞ is the long-term deflection calculated under the frequent load and a_1 is the deflection under the slab loading immediately before the installation of the finishes. The deflections a_∞ and a_1 should be calculated in accordance with M1 or M2 below as appropriate. If it is uncertain whether the slab will crack during construction, it is suggested that the incremental deflection is taken as the greatest of the values given by M1 and M2 below.

M1. No construction loading from casting slabs above: calculate a_∞ with the greatest of ζ_{perm} and ζ_{freq} and a_1 with the greatest of ζ_{strike} and ζ_1. Use the 28-day concrete tensile strength in the calculation of ζ_1, ζ_{perm} and ζ_{freq}. Take β as $0{\cdot}5$ in the calculation of ζ_{strike} and ζ_{perm}, $0{\cdot}7$ in ζ_{freq} and $1{\cdot}0$ in ζ_1.
M2. With construction loading: calculate a_1 with ζ^*_{peak} and a_∞ with ζ equal to the greatest of ζ^*_{peak}, ζ_{perm} or ζ_{freq}. Assume $\beta = 0{\cdot}7$ in the calculation of ζ_{peak} and ζ_{freq}.

The cracking moment M_{r} should be calculated with f_{ctstrike} in ζ_{strike} and f_{ctpeak} in ζ_{peak}. The cracking moment should be calculated with short-term section properties I_1 and x_1. Parametric studies show that M1 tends to give the greatest incremental deflection but the least total deflection.

Rationale of span-to-depth ratios

It is helpful to review the underlying rationale of controlling deflections with span-to-depth rules before making a detailed evaluation of the Eurocode 2[1] span-to-depth rules. The maximum permissible span to effective depth ratio corresponding to a deflection limit $\delta/L = \alpha$ can be expressed as

$$L/d = \alpha/(X d \Psi_{\text{m}}) \tag{12}$$

where Ψ_{m} is mean curvature which can be calculated with Equations 1 and 2, d is the effective depth, L is the maximum permissible span and X is a coefficient which relates the deflection to the loading arrangement and boundary conditions.

The product $d\Psi_m$ is readily shown to be independent of the section depth for specified values of the reinforcement index $\rho = A_s/bd$, d/h, reinforcement stress σ_s and concrete material properties because the curvatures in the uncracked and fully cracked sections are inversely proportional to the effective depth.

BS 8110: Part 2[12] gives values for X derived on the assumption that the flexural rigidity is uniform along the length of the member. The assumption of uniform rigidity is reasonable for uncracked members and cracked members in which cracking is distributed over a significant proportion of the span. Analysis shows, however, that X reduces significantly below the values given in BS 8110: Part 2[12] if the cracking is localised near mid-span as is the case when the peak moment is only slightly greater than the cracking moment. This effect is illustrated in Figure 3 in which X is plotted against the loading ratio w/w_u for the end span of a continuous slab for various reinforcement indices. X varies with the length of the span that is cracked, which depends on the shape of the bending moment diagram and the loading ratio $\lambda M_u/M_r$ where λM_u is the maximum span moment. $\lambda M_u/M_r$ is proportional to $\lambda(d/h)^2/(1{\cdot}3 - h/2000)$ for given values of ρ and f_{ck} if the effective concrete tensile strength is calculated with Equation 10 which implies that X and, hence, the permissible span-to-depth ratio depends on λ, d/h, h, ρ and f_{ck}. The influence of h is undesirable and can be excluded by calculating the effective concrete tensile strength with Equation 9. It follows that the effect of variations in λ, d/h, ρ and f_{ck} should be included in the calculation of permissible span-to-depth ratios.

Eurocode 2 span-to-depth rules

Eurocode 2[1] states that it is generally unnecessary to calculate deflections explicitly since deflection problems can be avoided by dimensioning members to comply with the span-to-depth ratios in the code. The span-to-depth rules given in Eurocode 2[1] were derived[13] by curve-fitting the results of a parametric study of a series of simply supported uniformly loaded members measuring 1000 mm wide by 300 mm deep. The effective depth was assumed to be $0{\cdot}9h$ where h is the total section depth. Eurocode 2 states that slabs dimensioned with the Eurocode 2 span-to-depth rules will satisfy both the commonly used overall deflection limit of $L/250$ and the active deflection limit of $L/500$. The code is inconsistent with the background document which states that the slenderness limits were calculated by limiting the total deflection to $L/250$, even though it was found that limiting the deflection after the construction of partitions to $L/500$ was more restrictive. The following additional assumptions were made in the parametric studies used in the derivation of the Eurocode 2 span-to-depth rules.[13]

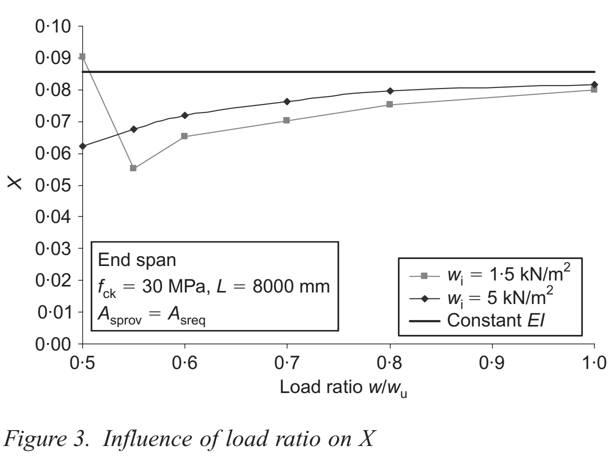


Figure 3. Influence of load ratio on X

(*a*) The mean 28-day direct concrete tensile strength was used in deflection calculations.
(*b*) The relative humidity was taken as 70%.
(*c*) The total characteristic load ($w_{tot} = w_d + w_i$) was assumed to equal $0{\cdot}71w_{uls}$, where w_{uls} is the design ultimate load calculated with load factors of 1·35 for dead loads and 1·5 for imposed loads. The total dead load w_d was made up of the self-weight w_{self} and the finishes w_{di} where $w_{self} = 0{\cdot}36w_{tot}$ and $w_{di} = 0{\cdot}24w_{tot}$. The self-weight w_{self} was applied at 10 days, the load of the finishes w_{di} was applied at 60 days and the permanent component of the imposed load $0{\cdot}3w_i$ was applied at 365 days.
(*d*) A short-term construction load equal to the permanent load of $w_d + 0{\cdot}3w_i$ was applied at striking.
(*e*) The characteristic yield strength of the reinforcement was taken as 500 MPa.
(*f*) The ratio between the permanent ($w_d + 0{\cdot}3w_i$) and design ultimate loads was assumed to be 0·5.

The span-to-depth ratios in Eurocode 2 are defined by Equation 13 below (equation 7·16 in Eurocode 2)

$$L/d_{basic} = K[11 + 1{\cdot}5\sqrt{f_{ck}}\rho_0/\rho + 3{\cdot}2\sqrt{f_{ck}}(\rho_0/\rho - 1)^{1{\cdot}5}] \text{ if } \rho \leqslant \rho_0 \tag{13a}$$

$$L/d_{basic} = K[11 + 1{\cdot}5\sqrt{f_{ck}}\rho_0/(\rho - \rho') + 1/12\sqrt{f_{ck}}\sqrt{(\rho'/\rho_0)}] \text{ if } \rho > \rho_0 \tag{13b}$$

where K accounts for structural form and is taken as 1 for simply supported spans, 1·3 for end spans of continuous spans, 1·5 for internal spans of continuous spans and 1·2 for flat slabs, ρ_0 is the reference reinforcement index $\sqrt{f_{ck}} \times 10^{-3}$, ρ is the required tension steel ratio at mid-span A_s/bd and ρ' is the required compression steel ratio at mid-span. Eurocode 2 states that $(L/d)_{basic}$

should be reduced by $7/L$ for spans greater than 7 m which support partitions likely to be damaged.

Eurocode 2 states that when other steel stress levels are used the basic span to effective depth ratios given by Equation 13 should be multiplied by $310/\sigma_s$ where σ_s is defined in the code as the tensile stress in the reinforcement at mid-span 'under the design load at SLS'. The code does not define what is meant by the design load at the serviceability limit state (SLS) but goes on to state that it is normally conservative to assume

$$Z = 310/\sigma_s = 500/(f_{yk}A_{sreq}/A_{sprov}) \tag{14}$$

where A_{sreq} is the area of flexural steel required for strength at mid-span and A_{sprov} is the area provided. The stress of 310 MPa in Equation 14 appears to have been calculated under the full service load (i.e. $g + q = 0{\cdot}71w_u$) since $0{\cdot}71 \times 500/1{\cdot}15 \sim 310$ MPa.

Alternatively the stress σ_s can be calculated as follows

$$\sigma_s = M(d - x_2)/I_2 \tag{15}$$

where M is the maximum span moment under the design service load, x_2 is the depth to the neutral axis of a fully cracked section and I_2 is the corresponding second moment of area. The present author considers that x_2 and I_2 should be calculated with E_c to be consistent with Equation 14. Eurocode 2 does not define the SLS loading case which should be used to calculate M. The author considers that M should be calculated under the total characteristic load $q_{tot} = g + q$ to be consistent with Equation 14.

Discussion of assumptions made in the derivation of the Eurocode 2 span-to-depth rules

The most contentious aspects of the derivation of the Eurocode 2 span-to-depth rules relate to the choices of the concrete tensile strength, loading ratio and the modification factor $310/\sigma_s$. The consequences of the assumptions made in the derivation of the Eurocode 2 span-to-depth rules are explored in the remainder of this paper.

Control of deflection by increasing A_{sprov}/A_{sreq}

In the UK, it is common practice to minimise slab thicknesses by increasing the area of flexural reinforcement provided in the span A_{sprov} to as much as twice that required for strength A_{sreq}. This practice frequently leads to significant economies in whole building costs since it reduces the building height and hence the area of external cladding. Increasing A_{sprov}/A_{sreq} reduces the service stress in the reinforcement and, hence, the deflection. Consideration of Equation 12 shows that increasing the area of reinforcement provided over that required for strength increases the permissible span of a given member under a specified loading ratio M/M_u (where M_u is the design ultimate moment) by a factor Ω equal to

$$\Omega = X_{ref}\Psi_{mref}/(X\Psi_m) \tag{16}$$

where the subscript 'ref' denotes the member with $A_{sprov} = A_{sreq}$. For given span L and loading ratio M/M_u, $X \sim X_{ref}$ as M_r/M is not changed significantly by the addition of surplus reinforcement to control deflection. Figure 4 shows that, in the long term, Ω can be approximated by

$$\Omega = (A_{sprov}/A_{sreq})^{0{\cdot}5} \tag{17}$$

Equation 17 is inconsistent with Eurocode 2 which states that 'where other stress levels are used the permissible span-to-depth ratios given by Equation 13 should be multiplied by $310/\sigma_s$'. The code goes on to say that $310/\sigma_s$ can be approximated with Equation 14 which implies that permissible span-to-depth ratios can be increased by a factor equal to A_{sprov}/A_{sreq} for grade 500 reinforcement. Equation 14 is flawed since the relationship between σ_s and curvature depends on whether σ_s changes as a result of (*a*) increasing A_{sprov}/A_{sreq} or (*b*) varying the loading ratio w/w_u. The inaccuracies in Equation 14 are recognised in the UK National Annex to Eurocode 2, which limits $310/\sigma_s$ to 1·5. Equation 17 is reasonably accurate unless the area of compression reinforcement is greater than A_{sreq} in which case it can be conservative.

Modification of Eurocode 2 span-to-depth rules to account for loading history and loading ratio

The Eurocode 2 span-to-depth ratios were derived for a loading ratio $w/w_u = 0{\cdot}5$ with $A_{sprov} = A_{sreq}$. In practice, w/w_u is typically around 0·6 for reinforced concrete slabs where self-weight can exceed $0{\cdot}5w_u$. The author considers that Equations 13 and 14 should be modified to account for variations in w/w_u from the value of 0·5 assumed in the derivation of Equation 13. Equation 13 needs to be modified to account for the loss in tension stiffening that occurs when the loading

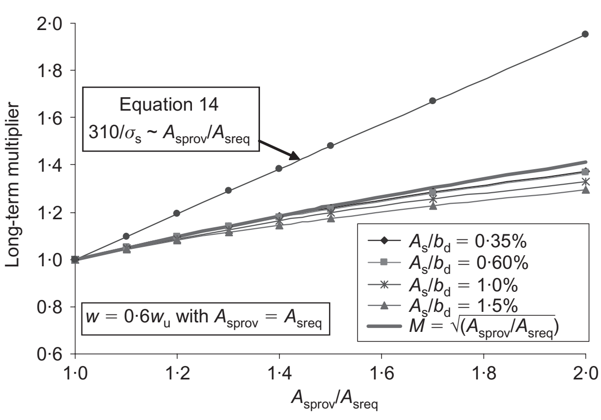


Figure 4. Influence of A_{sprov}/A_{sreq} on mean curvature

ratio w/w_u is increased above the value 0·5. Equation 14 needs to be modified since it is unable as it stands satisfactorily to account for variations in both A_{sprov}/A_{sreq} and w/w_u. Analysis shows that the loss of tension stiffening owing to loading ratios w/w_u greater than 0·5 can be included in Equation 13a if f_{ck} is replaced by an equivalent concrete strength $f_{ckequiv}$ given by

$$f_{ckequiv} = (f_{cteff}/0{\cdot}3)^{1{\cdot}5} \leqslant f_{ck} + 8 \tag{18}$$

where f_{cteff} is calculated as follows

$$f_{cteff} = (0{\cdot}9h/d)^2(0{\cdot}5w_u/w)f_{ctmod} \tag{19}$$

where w_u is the design ultimate load and

$$f_{ctmod} = \sqrt{2wk_{min}} \tag{20}$$

where k_{min} is the least of $\sqrt{0{\cdot}5f_{ctstrike}/w_{strike}}$, $\sqrt{0{\cdot}65f_{ctpeak}/w_{peak}}$ (where $\beta = 0{\cdot}65$ indirectly models the effect of ζ^*), $\sqrt{0{\cdot}5f_{ct28}/w_{perm}}$ and $\sqrt{0{\cdot}7f_{ct28}/w_{freq}}$. The term $(0{\cdot}9h/d)^2$ in Equation 19 accounts for the influence on the cracking moment of variations in h/d between the section under consideration and the reference section used in the derivation of the Eurocode 2 span-to-depth rules. Equation 19, which is only applicable to cracked slabs, modifies the concrete tensile strength to give the same interpolation coefficient ζ under the actual load w and $0{\cdot}5w_u$ used in the calibration of the Eurocode 2 span-to-depth rules. Analysis shows that deflections under w_{freq} are relatively insensitive to the concrete tensile strength when the reinforcement index is greater than ρ_o because the curvature tends towards that in a fully cracked section. It is therefore proposed that the actual concrete strength is used in Equation 13b and that the permissible span-to-depth ratio is taken as the greatest of the values given by Equations 13a with $f_{ckequiv}$ and 13b with f_{ck} if $\rho < \rho_o$ calculated with $f_{ckequiv}$. The coefficient ρ_o should be calculated with f_{cequiv} in Equation 13a and with f_{ck} in Equation 13b. The proposed procedure is illustrated in Figure 5. The critical span-to-depth ratio from Equation 13a (with $f_{ckequiv}$ from Equations 18 to 20) or Equation 13b (with f_{ck}) as appropriate should be multiplied by $0{\cdot}5w_u/w$ to account for the difference between the actual loading ratio and that assumed in the derivation of the Eurocode 2 span-to-depth rules.

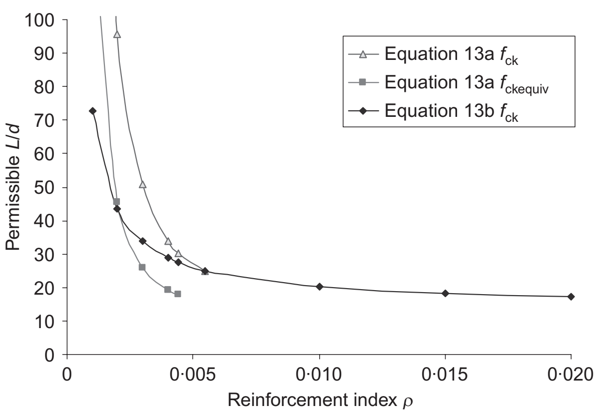


Figure 5. Influence of $f_{ckequiv}$ on $L/d_{permissible}$

Influence of span

Eurocode 2 requires the permissible span-to-depth ratio to be reduced by $7/L$ for spans greater than 7 m. The need for this reduction factor did not emerge in the calibration exercise described in the background document to the Eurocode 2 span-to-depth rules because

(*a*) the concrete tensile strength was taken as the splitting strength, which is independent of the section depth
(*b*) the effective depth was assumed to be a constant proportion of the section depth
(*c*) the loading ratio M/M_u was assumed to be 0·5
(*d*) no consideration was given to the 20 mm limit on incremental deflection.

The terms in Equation 12 for $L/d_{permissible}$ are independent of the slab thickness and hence span, for given f_{ck} and reinforcement index ρ, when assumptions (*a*) to (*c*) above apply. The term $7/L$ is needed in practice since assumptions (*a*) to (*c*) do not apply to practical slabs where (*a*) the cover is independent of span and (*b*) the loading ratio w/w_u increases with span for constant superimposed loads. The reduction factor $7/L$ is required to compensate for

(*a*) the reduction in $M_r/(0{\cdot}5M_u)$ with increasing span in members with constant cover and reinforcement index ρ (owing to the increase in d/h and f_{ct}/f_{fl})
(*b*) the increase in loading ratio w/w_u with span (owing to the increase in h) for slabs loaded with the same superimposed loads
(*c*) the requirement to limit incremental deflections to 20 mm.

The effect of (*a*) and (*b*) above is to reduce M_r/M which in turn increases the interpolation coefficient ζ in Equation 1 above that implicit in the Eurocode 2 span-to-depth rules. The reduction in M_r/M also increases the length of the span which is cracked which affects X in Equation 12 as shown in Figure 3. The increase in d/h with span is particularly significant in lightly reinforced members because it reduces the reinforcement ratio ρ required to resist M_u below that implicit in Equation 13, which was derived with $d/h = 0{\cdot}9$. The reduction in ρ leads to a significant increase in the permissible span-to-depth ratio given by Equation 13 which is offset by the $7/L$ factor in Eurocode 2.

Proposed modification to Eurocode 2 span-to-depth rules

It is recommended that permissible span-to-depth ratios are calculated as follows. If $\rho < \rho_{oequiv}$ take $L/$

$d_{\text{permissible}}$ as the greatest value given by either Equation 13a with f^*_{ckequiv} and $\rho_{\text{oequiv}} = 10^{-3}\sqrt{f^*_{\text{ckequiv}}}$ or Equation 13b evaluated with f_{ck} and $\rho_{\text{o}} = 10^{-3}\sqrt{f_{\text{ck}}}$ where

$$f^*_{\text{ckequiv}} = f_{\text{ckequiv}}[1 - 8(A_{\text{sprov}}/A_{\text{sreq}} - 1)/(f_{\text{ck}} + 8)] \tag{21}$$

where f_{ckequiv} is given by Equation 18 and $1 \leqslant A_{\text{sprov}}/A_{\text{sreq}} \leqslant 2$.

If $\rho \geqslant \rho_{\text{oequiv}}$ calculate $L/d_{\text{permissible}}$ with Equation 13b in conjunction with f_{ck} and $\rho_{\text{o}} = 10^{-3}\sqrt{f_{\text{ck}}}$.

The multiplier $310/\sigma_{\text{s}}$ in Eurocode 2 should be replaced by

$$Z = (0{\cdot}5w_{\text{u}}/w)(A_{\text{sprov}}/A_{\text{sreq}})^{0{\cdot}5} \tag{22}$$

where w is the load under which deflections are being limited (e.g. w_{freq}) and w_{u} is the design ultimate load.

A_{sreq} should be taken as the area of reinforcement required to resist the maximum design elastic bending moment in the span under all load combinations. Parametric studies show that w should not be taken as less than $0{\cdot}5w_{\text{u}}$ or greater than $0{\cdot}65w_{\text{u}}$ in Equation 19. The actual loading ratio w/w_{u} should be used in Equation 22. Equation 21 accounts for the increase in X (see Equation 16) relative to the value implicit in Equation 13a when the slab thickness is reduced by increasing $A_{\text{sprov}}/A_{\text{sreq}}$. The effect of Equation 21 is to reduce the 28-day concrete tensile strength from f_{ctmeff} (see Equation 9) to the value used in the calibration of Equation 13a when $A_{\text{sprov}}/A_{\text{sreq}} = 2$.

The total deflection in members sized with this method is close to span/250. It may be necessary to reduce the overall deflection to below span/250 to control the incremental deflection subsequent to fit-out to the lesser of span/500 or 20 mm. The incremental deflection Δa (see Equation 11) depends on (*a*) the span, (*b*) the ratio of the design service load (e.g. w_{freq}) to the load immediately before the installation of the finishes and (*c*) the peak construction load w_{peak}. Parametric studies show that when slabs are subject to peak construction loads of $0{\cdot}04h$ kN/m^2 the incremental deflection (Δa) under w_{freq} is typically less than 50% of the total deflection (a) if the peak construction load w_{peak} is similar to or greater than w_{freq}. Δa increases towards around $0{\cdot}7a$ if w_{freq} is greater than w_{peak}. The incremental deflection $\Delta a = \beta a$ can be limited to 20 mm by multiplying the span L corresponding to an overall deflection αL (e.g. $L/250$) by L^*/L where $L^* = 0{\cdot}02/(\alpha\beta)$. For example, $L^* = 10$ m as in BS 8110 if $\alpha = 1/250$ and $\beta = 0{\cdot}5$. $L^* = 7$ m as in Eurocode 2 if $\alpha = 1/250$ and $\beta = 0{\cdot}71$. It is suggested that L^* is taken as 10 m for members subject to construction loading from supporting the weight of wet concrete in slabs above. In the absence of construction loading, L^* can conservatively be taken as 7 m if there is a need to limit incremental deflections to 20 mm under w_{freq}.

Equation 21 is based on the assumption that the design ultimate load equals $1{\cdot}35g_{\text{k}} + 1{\cdot}5q_{\text{k}}$ as used in the derivation of the Eurocode 2 span-to-depth rules. If the load factor for dead load (g_{k}) is reduced to 1·25 in accordance with equation 6·10 in Eurocode 0, the loading ratio $0{\cdot}5w_{\text{u}}/w$ should be taken as $(1{\cdot}35g_{\text{k}} + 1{\cdot}5q_{\text{k}})/(1{\cdot}25g_{\text{k}} + 1{\cdot}5q_{\text{k}})\ 0{\cdot}5w_{\text{u}}/w$.

Evaluation of proposed modification to Eurocode 2 span-to-depth rules

The present author has carried out a comprehensive series of parametric studies to compare total and incremental deflections in continuous one-way spanning beams with depths equal to the minimum allowed by

M1. BS 8110[11]
M2. Eurocode 2 with $Z = (7/L)(A_{\text{sprov}}/A_{\text{sreq}})$ where $7/L \leqslant 1$
M3. Eurocode 2 as interpreted by the Concrete Centre[14] with $Z = (310/\sigma_{\text{s}} \leqslant 1{\cdot}5)7/L$ where $\sigma_{\text{s}} \sim (f_{\text{yk}}/\gamma_{\text{s}})[w_{\text{perm}}/w_{\text{u}}][A_{\text{s,req}}/A_{\text{s,prov}}]/\delta$ where $7/L \leqslant 1$
M4. Eurocode 2 as modified by the author.

Deflections were calculated with and without construction loading in a three-span beam of 1 m width which was continuous over simple supports with the moment at the external supports varying between $M = 0$ and $M = 0{\cdot}06FL$ (where $F = w_{\text{u}}L$) to simulate the restraint provided by supporting walls or columns. The span was varied between 6 m and 15 m, and the imposed load was varied between 1·5 and 50 kN/m^2 as noted. The superimposed dead load was taken as 1·5 kN/m^2 for imposed loads up to 7·5 kN/m^2. The total dead load was calculated in terms of an assumed load ratio $w_{\text{freq}}/w_{\text{u}} = 0{\cdot}6$ for beams carrying superimposed loads more than 7·5 kN/m^2 with 75% of the dead load assumed to be self-weight. The peak construction load was taken as $1{\cdot}6w_{\text{self}} = 0{\cdot}04h$ for slabs (where h is the slab thickness in mm). The concrete cylinder strength was varied between 30 and 50 MPa. Maximum and incremental deflections were calculated using the rigorous approach described in this paper in conjunction with the recommendations given in paragraphs M1 and M2 below Equation 11. Peak deflections were calculated with the external spans loaded with the frequent load and the internal spans loaded with the permanent load. The reinforcement in the spans A_{sreq} was designed to resist the maximum design moment under pattern loading, which was taken as $0{\cdot}086FL - 0{\cdot}45M_{\text{ext}}$ in the external spans. The coefficient K in Equation 13 was increased as follows in M4 to take account of the beneficial effect of restraint at the external supports

$$K = 1{\cdot}3 + 0{\cdot}2M_{\text{ext}}/(0{\cdot}04FL) \leqslant 1{\cdot}5 \tag{23}$$

where M_{ext} is the moment at the external support.

Figures 6 to 8 compare the member thicknesses given by methods 1 to 4 for various scenarios. It is

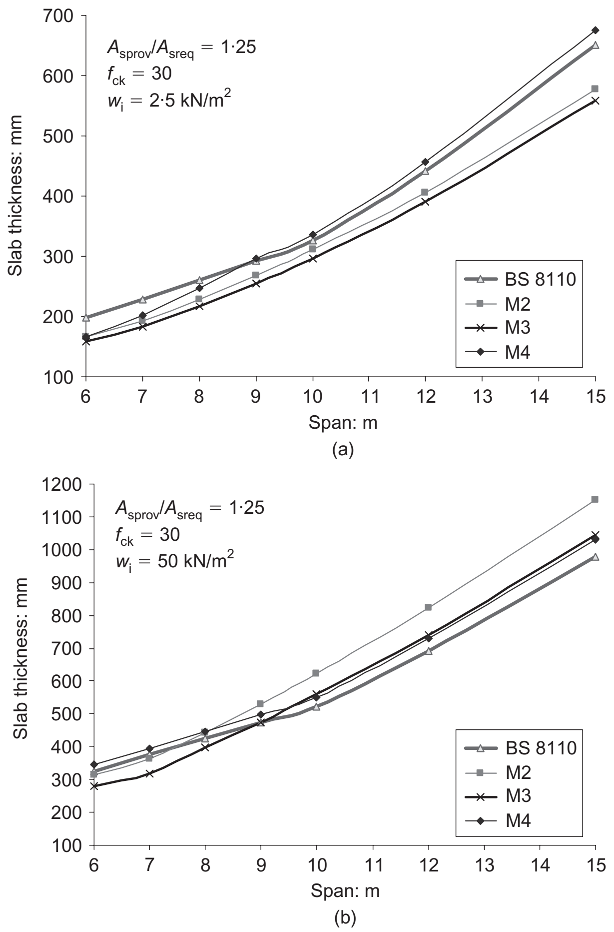


Figure 6. Influence of span on slab depths for $M_{ext} = 0{\cdot}04FL$, $A_{sprov}/A_{sreq} = 1{\cdot}25$ with w_i = (a) 2·5 and (b) 50 kN/m²

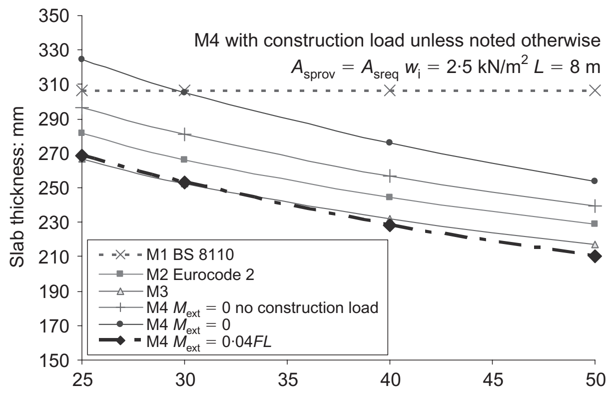


Figure 7. Influence of concrete strength on slab thickness ($M_{ext} = 0$ unless noted otherwise)

interesting to note that the proposed method M4 gives similar member thicknesses to BS 8110 for spans $\geqslant 10$ m with $f_{ck} = 30$ MPa. M2 (Eurocode 2) tends to give thinner slabs than M4 but deeper beams due to the use of the factor 7/L in M2 compared with $10/L$ in M4 and BS 8110. The slab thickness is independent of the

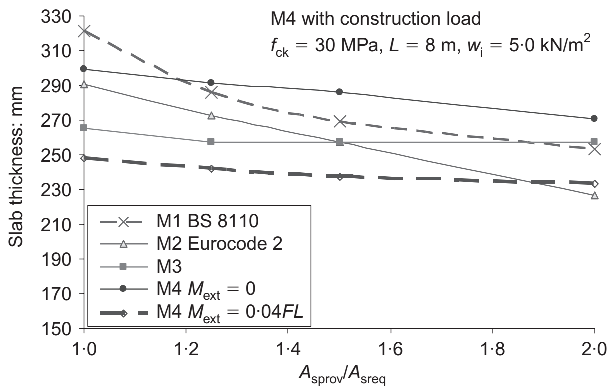


Figure 8. Influence of A_{sprov}/A_{sreq} on slab thickness ($M_{ext} = 0$ unless noted otherwise)

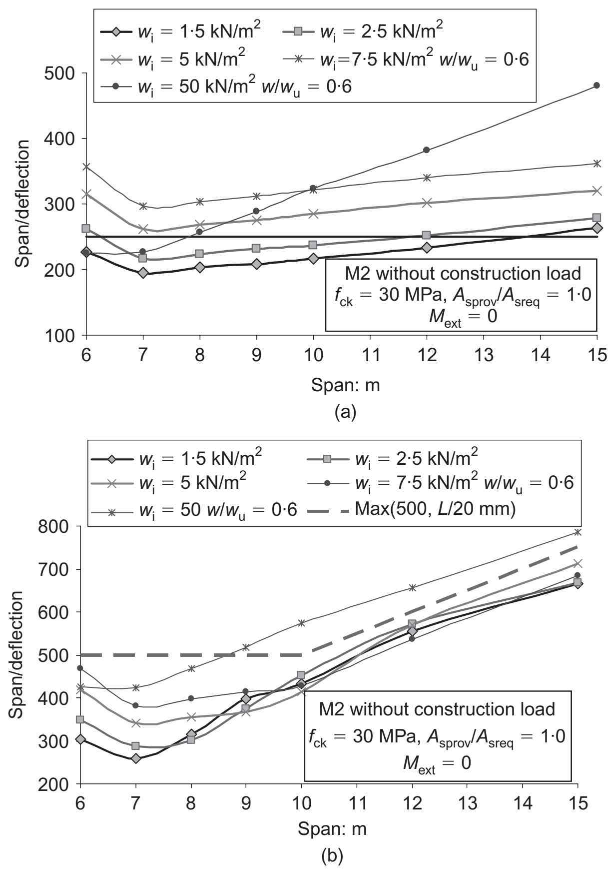
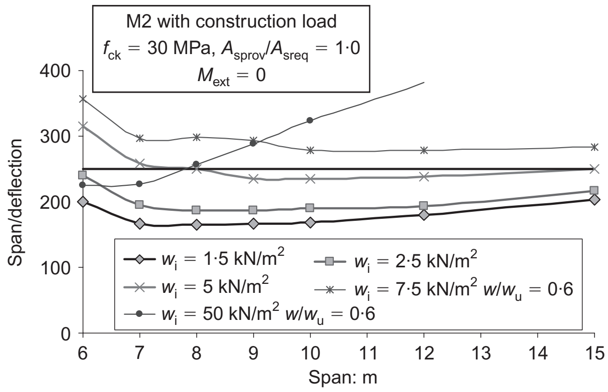
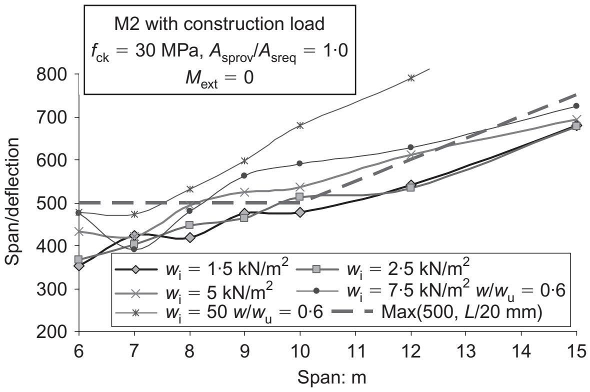


Figure 9. M2 (Eurocode 2) Deflections in end span calculated without construction load ($M = 0$ at external support): (a) total deflection; (b) incremental deflection

construction load in M1 to M3 above but not M4. Figures 9, 10 and 13b show deflections can significantly exceed code limits in slabs dimensioned with the Eurocode 2 span-to-depth rules particularly when the slab thickness is reduced by increasing A_{sprov}/A_{sreq}. The main reason for the relatively large deflections in Figures 9 and 10 is that an unrealistically low loading ratio of $w/w_u = 0{\cdot}5$ was assumed in the derivation of the Eurocode 2 span-to-depth rules. In practice, deflections in simply supported members are likely to be signifi-

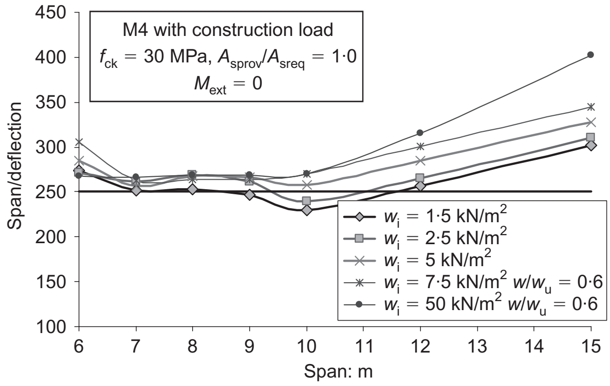


(a)

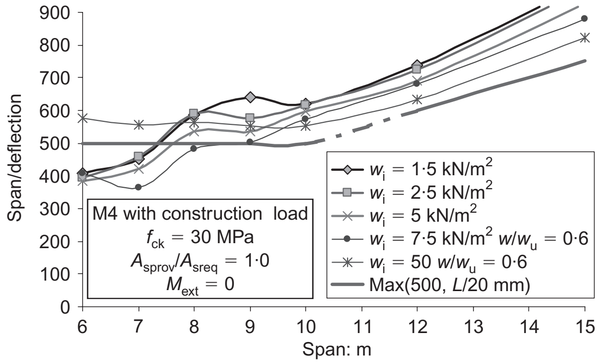


(b)

Figure 10. M2 (Eurocode 2) Deflections in end span calculated with construction load ($M = 0$ at external support): (a) total deflection; (b) incremental deflection



(a)



(b)

Figures 11. M4: Deflections in end span calculated with construction load ($M = 0$ at external support): (a) total deflection; (b) incremental deflection

cantly less than shown in Figures 9 and 10 since supports are seldom truly simply supported in reality as noted by Beeby[15] in his justification of the span-to-depth rules used in BS 8110. Comparison of Figures 9 and 10 shows that construction loading tends to be beneficial since it reduces the incremental deflection seen by partitions and finishes.

Figures 11 and 12 show that the deflections are close to the code limits of span/250 for total deflections and the least of span/500 or 20 mm for incremental deflections in slabs designed with M4. Figure 13 shows that all the methods tend to overestimate the reduction in slab thickness which can be obtained by adding surplus flexural reinforcement. Figure 14 compares deflections calculated in members sized with M2 to M4 with restraining moments varying between 0 and 0·06FL at the external supports for a span of 8 m, with $f_{ck} = 30$ MPa and $A_{sprov} = 1{\cdot}25A_{sreq}$. Figure 14c shows that M4 gives deflections close to span/250 for restraining moments varying between 0 and 0·04FL. Figure 14d shows that the limiting deflection can be increased to span/200 by multiplying the span-to-depth ratios given by M4 by 250/200 = 1·25.

Figures 6 to 8 show that M3[14] tends to give the thinnest slabs and consequently highest deflections. Deflections were calculated in slabs designed with M3 assuming that there was no moment at the external support. The deflections were calculated with construction loading assuming that in reality there was a restraining moment of 0·04FL (which is the moment given in Table 3·12 of BS 8110 for continuous end supports) at the external support. Figure 15 shows that the resulting deflections were close to the Eurocode 2 limits and suggests that slab thicknesses given by M3 are justifiable and economic if additional moment restraint is present in reality to that assumed in the design of the flexural reinforcement in the span. If this approach is adopted, continuity moments should be taken into account in the design of flexural reinforcement at external supports, columns and punching shear reinforcement.

Conclusions

Eurocode 2 allows significant reductions in slab thickness compared with BS 8110[11] since it accounts for the actual concrete tensile strength in deflection calculations. The drawback is that deflections depend significantly on material properties and loading, all of which are usually uncertain. This paper proposes a standard procedure for calculating deflections with Eurocode 2 that can be used in the absence of better information. Further research is required to determine how deflections in the field compare with deflections

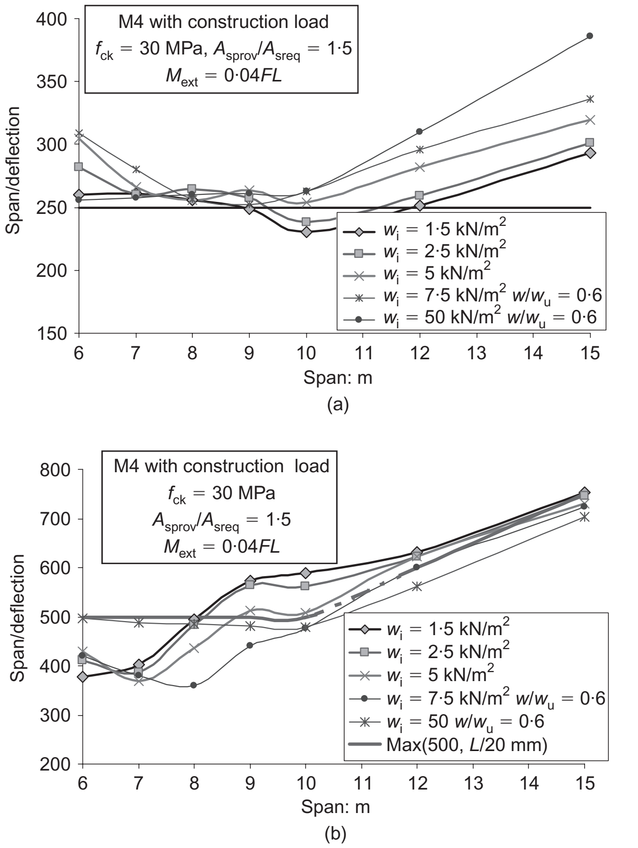
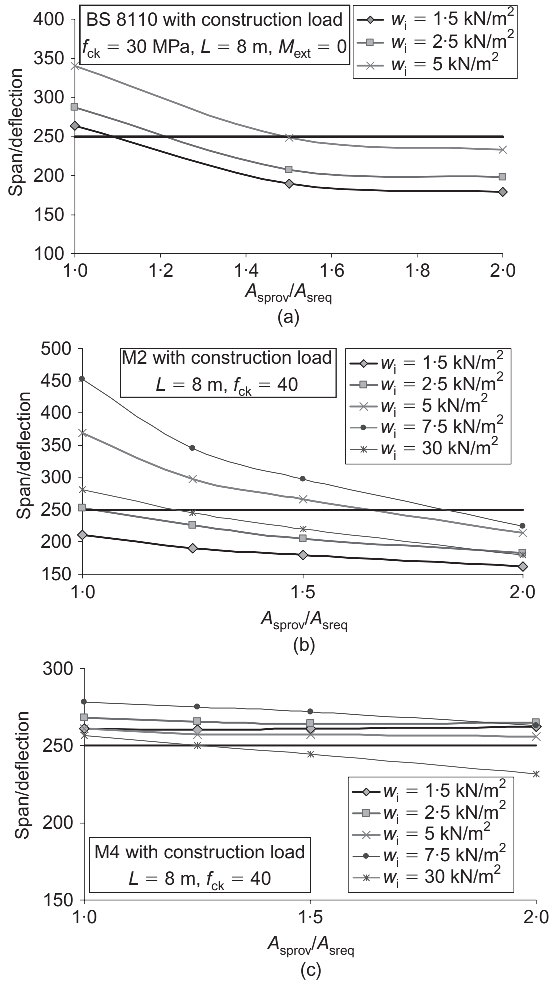
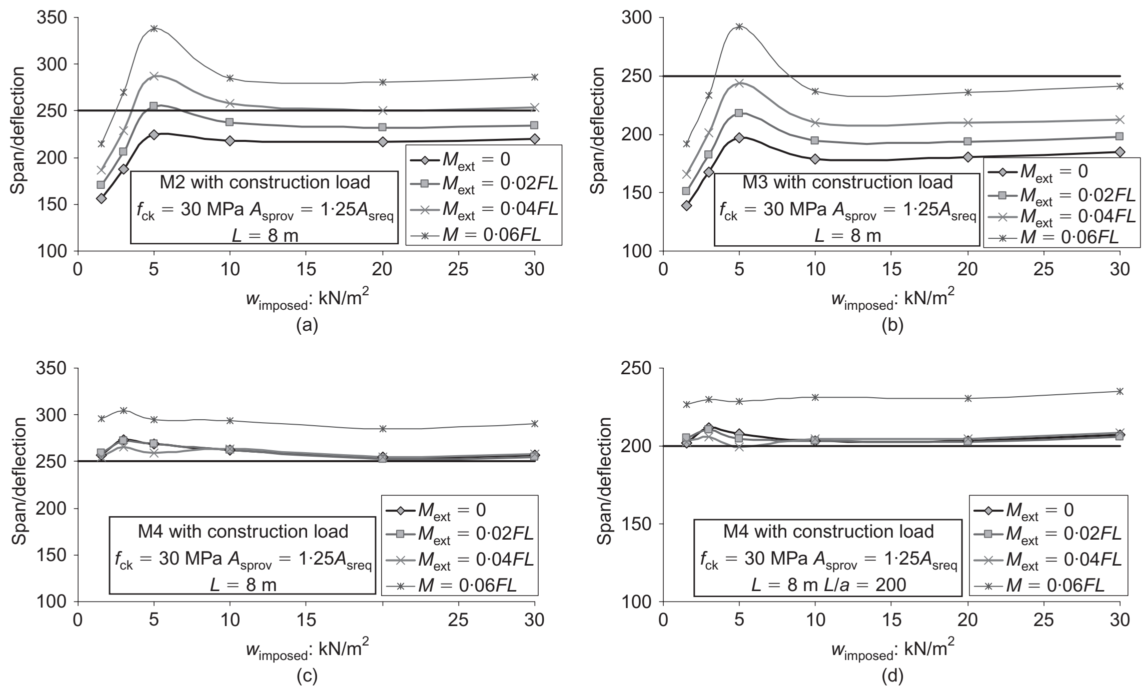


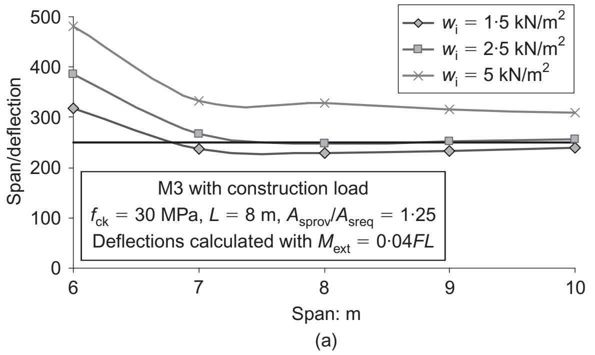
Figure 12. M4: deflections in end span calculated with construction load, A_{sprov}/A_{sreq} = 1·5 and M = 0·04FL at external support: (a) total deflection; (b) incremental deflection



Figures 13(a) to (c). M4: influence of A_{sprov}/A_{sreq} on deflections in end span of slabs dimensioned with BS 8110, M2 and M4



Figures 14(a) to (d). Influence of end restraining moment on total deflections calculated in slabs dimensioned with methods 2 to 4 with A_{sprov}/A_{sreq} = 1. 25



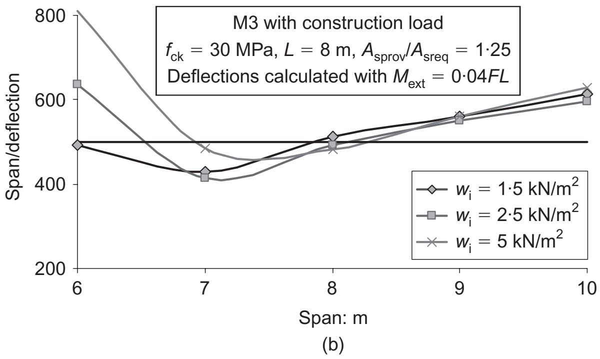


Figure 15(a) to (b). M3: deflections in end span with construction loading and $M_{ext} = 0{\cdot}04FL$ for slab designed as simply supported at end support

calculated with the proposed procedure. It is shown that the Eurocode 2 span-to-depth rules do not account for increases in deflection arising from cracking during construction and variations in d/h and loading ratio w/w_u. A modification is proposed to the Eurocode 2 span-to-depth ratios (M4) which rationally accounts for these factors. Figure 13a shows that deflections can exceed BS 8110 and Eurocode 2 limits in slabs designed in accordance with BS 8110 and raises the question of whether the deflection limits in Eurocode 2 are too onerous. Comparison with BS 5950[16] shows that the deflection limits in Eurocode 2 are very onerous compared with those used for steel construction. There seems to be a good case for increasing the incremental deflection limit in Eurocode 2 to span/360 for brittle partitions, to bring it in line with current practice for structural steelwork,[16] but not more than 20 mm. It is also suggested that the overall deflection limit can be increased to span/200 where the slab is hidden by raised floors and ceilings. Careful judgement needs to be exercised in the choice of member depths and deflection limits. In many cases, it may be sufficient to limit the increment in deflection following fit-out to say 20 mm. In this, case member depths can often be reduced to those given by M3,[14] particularly if additional restraint is present to that assumed in the design of flexural reinforcement. M2 gives reasonable results with the exception of lightly loaded slabs with minimal end restraint if A_{sprov}/A_{sreq} is limited to 1·5 as in the UK National Annex to Eurocode 2. M4, which is easily incorporated into a spreadsheet, is advantageous since it relates member thickness to prescribed deflection limits taking due account of variations in the loading ratio w/w_u and rotational restraint at end supports.

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Discussion contributions on this paper should reach the editor by 1 February 2010