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Development of Statistical Quality Assurance Criterion for Concrete Using Ultrasonic Pulse Velocity Method

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The ultrasonic pulse velocity (UPV) test has a strong potential to be developed into a very useful and relatively inexpensive in place test for assuring the quality of concrete placed in a structure. The main problem in realizing this potential is that the relationship between compressive strength and UPV is uncertain and concrete is an inherently variable material. This paper presents a probabilistic framework to incorporate these uncertainties rationally and systematically. In doing so, a reliable and consistent statistical quality assurance criterion based on UPV measurements can be developed. The proposed method of analyzing UPV data allows the engineer to make quantifiable conclusions regarding the quality of concrete in question. In addition, the proposed method also allows the engineer to strike an equitable balance between acceptable risk and cost of remedial measures.

Keywords: compressive strength; quality assurance; ultrasonic pulse velocity tests.

INTRODUCTION

Besides satisfying the strength requirement based on the cube or cylinder test, the concrete placed in a structure must be of uniform quality, and free of voids and discontinuities, especially honeycombing. Lack of sufficient attention to handling and placing of concrete can result in poor quality concrete in the structure, even if good quality ready-mixed concrete is used. When a dispute arises regarding the quality of concrete in a structural member, tests are carried out to affirm the strength of the concrete in question or to investigate areas of poor consolidation and voids. Currently, coring for samples to undergo compressive load testing is widely adopted in the construction industry. However, it is costly and time-consuming to carry out coring. In addition, there is a practical limit to how many samples can be taken from a structural element without compromising its integrity. Because of the small sample size, it is generally quite difficult to draw reliable or statistically meaningful conclusions. There is also the problem of having to repair the damage done to the structure due to coring, which involves additional costs.

There is an obvious need for a reliable nondestructive test method to complement or replace the existing destructive means of verifying the strength of in place concrete. The ultrasonic pulse velocity (UPV) test has this potential because it is entirely nondestructive in nature, simple to operate, and relatively inexpensive. However, the UPV test is currently not widely used as a primary test for concrete quality. It is used, in most instances, as a supplementary indicator of the quality of concrete in a structure. This is because the relationship between the compressive strength of concrete and UPV is not simple.

Over the years, considerable research[1-9] has been undertaken to study this relationship. Sturrup et al.[2] observed a relationship between UPV and the logarithm of the compressive strength. Ben-Zeitun[3] and Price and Hynes,[4] on the other hand, proposed a linear relationship between UPV and compressive strength, while Poh[5] suggested that the relationship is exponential. Regardless of the functional form, it is widely recognized that the relationship is not unique, but is affected by numerous factors such as the properties and proportion of the constituent materials,[2] age of concrete,[3] presence of microcracks,[4] moisture content,[6] and stresses in the concrete specimens.[7] In addition to the variability in the relationship, it is also recognized that factors such as surface condition, temperature of concrete, path length, and shape and size of specimen can introduce extraneous variability to UPV measurements if they are not properly controlled.[8] The presence of reinforcements also affects UPV measurements.[10] In addition, UPV measurements taken from different specimens are inherently variable because concrete is a heterogeneous material. The overall velocity of the ultrasonic pulse passing through aggregates, hydration products, and void space may be slightly different depending on their proportion along the path of wave propagation. The inherent variability of UPV measurements generally cannot be removed, although it can be reduced to some extent by proper mixing prior to casting.

In view of the prevously mentioned uncertainties and the associated difficulties in interpretation, the application of the UPV test to quality assurance in practice is currently limited.[10] None of these studies considered uncertainties systematically in the development of a correlation between compressive strength and UPV, nor did they demonstrate how the relationship could be reliably and consistently applied to quality assurance. Engineers are therefore uncomfortable with the use of UPV as a primary method of assessing concrete quality, which prevents the advantages of UPV from being exploited fully. If a reliable quality assurance criterion based on UPV is available, a larger part of the structure can be tested quickly and cheaply to supplement the limited test results obtained from destructive testing. This will unquestionably become a valuable in place method that can contribute significantly to the overall quality assurance programs in large projects.

RESEARCH SIGNIFICANCE

The aim of this study is to make UPV the primary method of assessing the quality of in place concrete while coring, with its inherent disadvantages, would be relegated to the role of calibrating the UPV with compressive strength and

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confirming some of the locations of honeycombing and poor concrete strength identified by UPV tests. The ability to test many locations using UPV and make a rational decision based on a trade-off between properly quantified risk and cost is the key advantage to adopting a statistical quality assurance approach. In this paper, a probabilistic framework that incorporates uncertainties rationally and systematically will be developed to illustrate how a consistent statistical quality assurance criterion can be derived based on UPV measurements. The framework is general and can be applied to any problem where indirect measurements (e.g., nondestructive test results) are used to diagnose some conditions in the concrete medium (e.g., weak strength, honeycombing) that are not readily amenable to direct examination. It will be shown that this probabilistic approach is a natural extension of the characteristic strength concept that is already widely used in practice. The proposed method of analyzing UPV data allows the engineer to make quantifiable conclusions regarding the quality of concrete in question. In addition, the proposed method also allows the engineer to strike an equitable balance between acceptable risk and cost of remedial measures.

TEST PROGRAM

The test program was primarily designed to obtain: a) an accurate probabilistic characterization of the UPV and the compressive strength of concrete; b) a high-quality statistical correlation between UPV and compressive strength at saturated surface dry condition; and c) an accurate probabilistic characterization of the prediction errors inherent in the correlation between UPV and compressive strength. To obtain a realistic assessment of concrete variability under field conditions, the concrete used in this study was site-bound concrete supplied by a local ready-mixed concrete supplier in Singapore. Three batches of concrete in grades of 35, 55, and 75 were tested. Table 1 summarizes the mixture proportion used in the three mixtures. The aggregates used for the concrete mixtures were river sand and crush granite, with maximum aggregate sizes of 5 and 20 mm, fineness moduli of 3.1 and 6.5, and specific gravities of 2.60 and 2.65, respectively. Superplasticizer was used to control the slump values of these concrete mixtures within the range of 125 ± 25 mm.

For each batch of concrete, 50 150 x 150 x 150 mm cube specimens were cast for statistical study. All the specimens were subjected to nondestructive UPV tests followed by destructive compressive strength tests on the 28th day. The equipment used in this study to measure the UPV is the portable ultrasonic nondestructive digital indicating tester (PUNDIT) with transducers operating at a frequency of 150 kHz.

Quality control

Because of the statistical nature of this study, stringent quality control had to be maintained throughout the test program to minimize the occurrence of systematic errors and the introduction of extraneous random errors. For example, all the cube specimens were water cured to minimize the variability in the moisture content, which can seriously affect the UPV readings.[3,8] Other controls included the constant calibration of the UPV meter, especially at the start and end of each session when UPV measurements were taken.

The ultrasonic pulse passing through the concrete was generated by a transducer serving as a transmitter held close to the surface of the concrete specimen. The onset of the pulse on the opposite face of the specimen was detected by another transducer that served as a receiver. This direct transmission method was adopted in this study because the maximum energy of the pulse was directed at the receiver and the noise to signal ratio could be minimized. Repeated measurements were taken at each transducer arrangement so that the consistency and accuracy of the UPV measurements were assured. The transmitter and receiver locations were also carefully marked so that repeated measurements could be taken from the same place.

To ensure good acoustic contact, all cube specimens were cast using smooth steel molds so as to obtain level surfaces. Acoustic contact was further improved with the use of acoustic grease. The layer of grease between the transducer and concrete surface was kept as thin as possible so that the readings taken would not be too adversely affected by the thickness of the grease. Furthermore, constant pressure was applied on the transducers during the course of the UPV measurements. By taking these precautions, extraneous variabilities introduced by poor procedural and operator controls were thus reduced.

Table 1—Summary of ready-mixed concrete mixture proportioning

Concrete composition	Grade of concrete, MPa 35	55	75
Cement, kg/m^3	390	470	520
Water, kg/m^3	175	169	160
Fine aggregate, kg/m^3	764	698	620
Coarse aggregate, kg/m^3	1010	1020	1050
Water-cement ratio, %	45	36	31
Percentage of fine aggregate, %	43.1	40.6	37.1

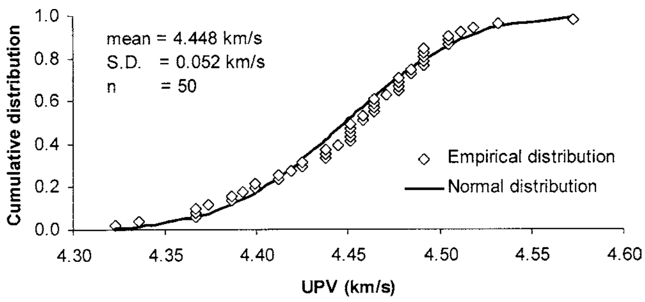
Table 2—Second-moment statistics of ultrasonic pulse velocity (UPV) and compressive strength of concrete

Variable	Concrete grade, MPa	Mean	Standard deviation	Coefficient of variation, %
UPV	35	4.448 km/s	0.052 km/s	1.17
	55	4.591 km/s	0.054 km/s	1.18
	75	4.662 km/s	0.052 km/s	1.12
Compressive strength	35	41.75 MPa	1.74 MPa	4.17
	55	63.11 MPa	2.93 MPa	4.64
	75	85.17 MPa	4.16 MPa	4.88

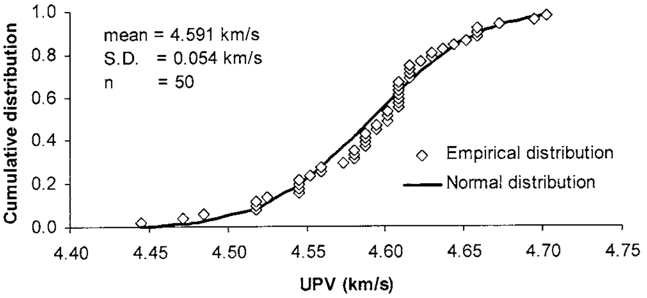
TEST RESULTS

Second-moment statistics

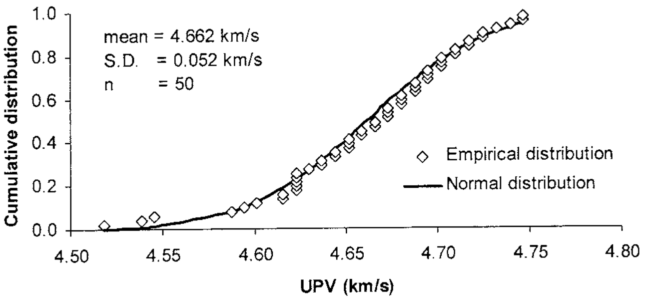
The mean, standard deviation, coefficient of variation of UPV, and compressive strength are shown in Table 2. The coefficient of variation is a measure of relative dispersion and is computed by dividing the standard deviation by the mean. The relatively low coefficients of variation for UPV (≈ 1.1 to 1.2%) and compressive strength (≈ 4.2 to 4.9%) are



(a) Grade 35

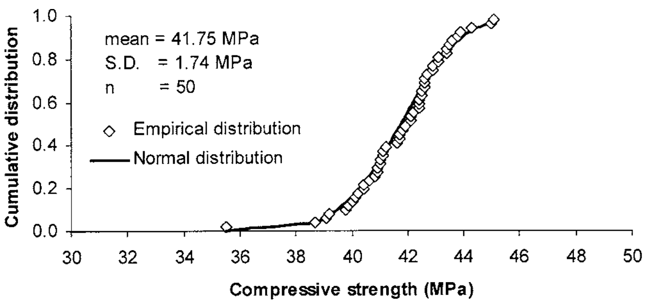


(b) Grade 55

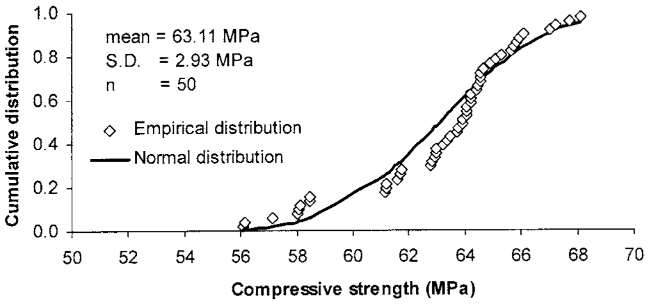


(c) Grade 75

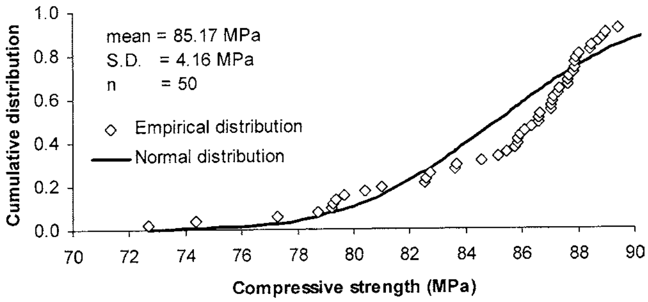
Fig. 1—Cumulative distribution functions of ultrasonic pulse velocity (UPV): (a) Grade 35; (b) Grade 55; and (c) Grade 75.



(a) Grade 35



(b) Grade 55



(c) Grade 75

Fig. 2—Cumulative distribution functions of compressive strength of concrete: (a) Grade 35; (b) Grade 55; and (c) Grade 75.

indications of the stringent quality control maintained throughout the test program. The corresponding coefficients of variation for in place concrete are likely to be higher than those obtained in this study. It is also worth noting that the coefficients of variation are relatively uniform across different grades of concrete.

Cumulative distribution functions

The cumulative distribution functions of the UPV and compressive strength for different grades of concrete are plotted and tested for normality using the standard Kolmogorov-Smirnov goodness-of-fit test. Figures 1 and 2, respectively, show that the empirical cumulative distribution functions of UPV and compressive strength can be approximately fitted to the theoretical normal cumulative distribution function. Results from the Kolmogorov-Smirnov test shown in Table 3 also indicate that the observed *D*-statistic is less than the critical value corresponding to the customary 5% level of significance. The null hypothesis that the data belong to a normal probability distribution therefore cannot be rejected at the 5% level of significance. Both UPV and compressive strength can thus be adequately modeled as normal random variables with the respective means and standard deviations indicated in Table 2.

STATISTICAL QUALITY ASSURANCE CRITERION

Statistical correlation between UPV and compressive strength

The relationship between UPV and compressive strength is shown in Fig. 3. It is observed that the compressive strength of the concrete specimens generally increases linearly with UPV. Such a linear relationship was also observed by a number of other researchers.[3,4] The relationship between UPV and compressive strength can be estimated using the standard linear regression technique. The best fit line for the data obtained in this study is

$$Y = 142.4X - 587.0 \tag{1a}$$

where Y = compressive strength (MPa) and X = UPV (km/s). Because of the large number of specimens tested in this

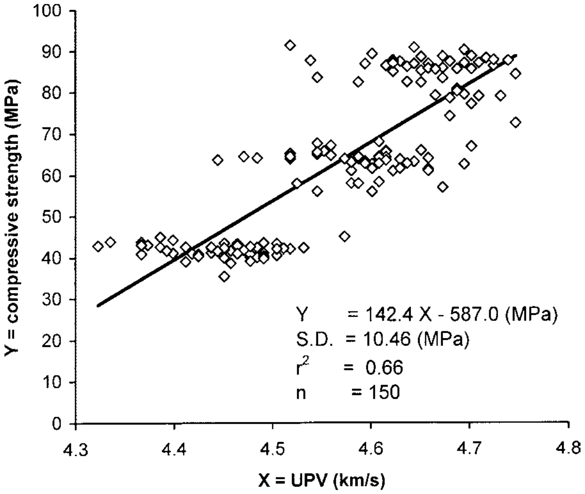


Fig. 3—Statistical correlation between UPV and compressive strength of concrete.

Table 3—Kolmogorov-Smirnov goodness-of-fit test for normal distribution

Variable	Grade of concrete, MPa	Observed *D*-statistic	Critical value $D_{5\%}$	Probability distribution
UPV	35	0.108	0.192	Normal
	55	0.104	0.192	Normal
	75	0.073	0.192	Normal
Compressive strength	35	0.076	0.192	Normal
	55	0.157	0.192	Normal
	75	0.181	0.192	Normal

Note: D-statistic = maximum absolute difference between empirical and theoretical cumulative distribution function. The null hypothesis that variable is normally distributed can not be rejected if observed *D*-statistic is less than critical value $D_{5\%}$.

study ($n = 150$), it is also possible to study the reliability of using Eq. (1) to predict compressive strength from UPV readings. The prediction errors are summarized in the form of a cumulative distribution function as shown in Fig. 4. The mean and standard deviation (SD) of the prediction errors are 0 and 10.46 MPa, respectively. The significant errors computed are not surprising because concrete is a heterogeneous material with considerable inherent variability, and UPV is not affected by compressive strength alone, as discussed previously.

As shown in Fig. 4, the empirical cumulative distribution function of the prediction errors is very similar to the theoretical normal case. The observed *D*-statistic from the Kolmogorov-Smirnov test is 0.089, which is smaller than the critical value of 0.11. The null hypothesis that the data belong to a normal probability distribution therefore cannot be rejected at the 5% level of significance. Based on the previously cited detailed error analysis, Eq. (1) can now be reformulated more realistically in probabilistic terms to account for the prediction errors as follows

$$Y = 142.4X - 587.0 + \varepsilon \tag{2a}$$

where ε = normal random variable with mean = 0, and standard deviation = 10.46 MPa.

Probabilistic model for quality assurance

To account for the variation in the compressive strength taken from different cube or cylinder specimens, it is customary to use the characteristic strength of concrete rather than the mean strength. The characteristic strength of concrete is defined in probabilistic terms as follows

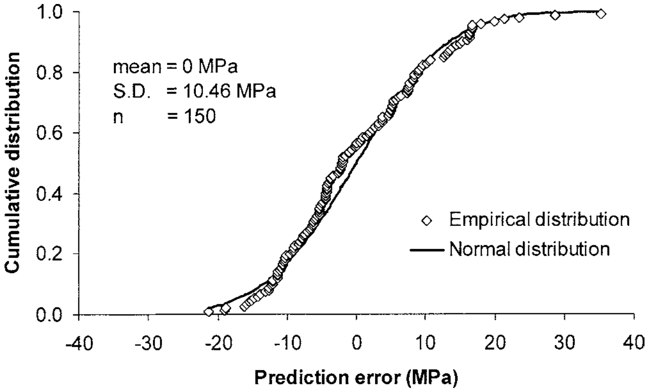


Fig. 4—Cumulative distribution function of prediction errors from linear regression.

$$P(Y < y_c) = \alpha \tag{3a}$$

where

Y = compressive strength;

y_c = characteristic strength; and

α = probability that the compressive strength of concrete is less than y_c.

The characteristic strength is therefore specified such that the fraction of concrete specimens α with compressive strength less than y_c is acceptably small. The commonly accepted value of α is 5%.[11] For compressive strength that is determined by direct destructive testing of cube or cylinder specimens, the existing practice on quality assurance [Eq. (3)] is already based on probabilistic principles that can rationally account for the variability in the test results.

In the case of the nondestructive UPV test, a characteristic value for UPV can be defined in an analogous manner as follows

$$P(X < x_c) = \beta \tag{4a}$$

where

X = UPV;

x_c = characteristic UPV; and

β = probability that the UPV of concrete is less than x_c.

From a practical engineering point of view, the evaluation of the characteristic value of UPV x_c is, however, not very useful or meaningful unless it can be related to the characteristic strength y_c.

To derive this relationship, the previously established linear relationship between UPV and compressive strength is first expressed as

$$Y = aX + b + \varepsilon \tag{5a}$$

where

a = 142.4 (MPa s/km);

b = –587.0 (MPa); and

ε = normally distributed random variable with mean = 0 and standard deviation = 10.46 MPa.

The model parameters a and b may change depending on factors such as the type of aggregates used in the concrete. In practice, the actual relationship between UPV and compressive strength should be calibrated using some representative core samples. Based on the prediction errors observed in this study, it is estimated that approximately 10 samples are needed to obtain a reasonably accurate mean prediction. It may be possible to augment this sample size with the cube or cylinder specimens cast on site for quality control or laboratory specimens constituted using the same concrete mixture to obtain a more refined mean relationship and a better estimate of the uncertainty about the mean σ_ε.

It was demonstrated previously that the UPV can be adequately modeled as a normal random variable with mean = μ_X and standard deviation = σ_X. Because of the normality of X and ε and the additive form of Eq. (5), it can be easily shown that the compressive strength Y also follows a normal distribution with mean μ_Y and standard deviation σ_Y given by

$$\mu_Y = a\mu_X + b \tag{6a}$$

$$\sigma_Y^2 = a^2\sigma_X^2 + \sigma_\varepsilon^2 \tag{6b}$$

Note that the deduced normality of the compressive strength is also consistent with the experimental data shown in Fig. 2.

The relationship between the characteristic strength y_c and the characteristic UPV x_c can now be established by rewriting Eq. (3) as follows

$$P(aX + b + \varepsilon < y_c) = \alpha \tag{7a}$$

Since $Y = aX + b + \varepsilon$ is a normal random variable, Eq. (7) can be readily calculated as follows

$$\Phi\left(\frac{y_c - \mu_Y}{\sigma_Y}\right) = \Phi\left(\frac{y_c - a\mu_X - b}{\sqrt{a^2\sigma_X^2 + \sigma_\varepsilon^2}}\right) = \alpha \tag{8a}$$

where $\Phi(\bullet)$ = cumulative distribution function of a standard normal variate that can be readily obtained from standard statistical tables.[12] The mean UPV μ_X is thus related to the characteristic strength as

$$y_c = \Phi^{-1}(\alpha)\sqrt{a^2\sigma_X^2 + \sigma_\varepsilon^2} + a\mu_X + b \tag{9a}$$

Finally, note that X is also normally distributed and therefore Eq. (4) can be evaluated as

$$\Phi\left(\frac{x_c - \mu_X}{\sigma_X}\right) = \beta \tag{10a}$$

or

$$x_c = \mu_X[1 + \Phi^{-1}(\beta)\delta_X] \tag{11a}$$

where

$\delta_X = \sigma_X/\mu_X$ = coefficient of variation of X.

Note that the values of δ_X obtained in this study are very uniform (Table 2). Therefore, in practice, it may be sufficient to determine μ_X only, which typically requires a smaller sample size, while σ_X is simply evaluated as $\mu_X\delta_X$. By combining Eq. (9) and (11), the relationship between y_c and x_c can be expressed as follows

$$y_c = \Phi^{-1}(\alpha)\sqrt{a^2\sigma_X^2 + \sigma_\varepsilon^2} + ax_c[1 + \Phi^{-1}(\beta)\delta_X]^{-1} + b \tag{12a}$$

Either Eq. (9) or (12) can be used for quality assurance. Eq. (12) is, however, slightly more robust in the sense that it is less affected by changes in δ_X. For example, assume that $\alpha = \beta = 0.05$, a = 142.4 MPa s/km, b = –587.0 MPa, σ_ε = 10.46 MPa, and x_c = 4.422 km/s. For values of δ_X ranging from 0.01 to 0.02, the value of y_c determined from Eq. (12) varies from 33.1 to 36.6 MPa. The mean UPV (μ_X) corresponding to the above characteristic UPV is 4.534 km/s [evaluated using Eq. (11) with an average δ_X of 0.015]. For the same variation in δ_X, the value of y_c determined from Eq. (9) now varies over a larger range from 31.3 to 38.4 MPa.

Summary of proposed approach

The proposed approach for quality assurance using UPV can be summarized as follows:

1. Determine the model parameters a and b in Eq. (5) by calibrating the relationship between UPV and compressive strength using some representative core samples. If possible, augment this sample size with the cube or cylinder specimens cast on-site for standard compressive strength tests or laboratory specimens constituted using the same concrete mixture. In the absence of additional information, a first-order estimate of the uncertainty about the mean relationship σ_ε is 10 MPa.

2. Measure the UPV of the concrete section in question and summarized the results in the form of the mean μ_X and standard deviation σ_X as follows

$$\mu_X = \frac{1}{n}\sum_{i=1}^{n} x_i \tag{13a}$$

$$\sigma_X = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(x_i - \mu_X)^2} \tag{13b}$$

where

n = number of UPV readings; and

x_i = i-th UPV reading.

3. Select an acceptable risk level α and evaluate the characteristic strength y_c corresponding to the UPV measurements using Eq. (9).

4. Alternatively, first evaluate the characteristic UPV value x_c using Eq. (11), with β generally taken as 0.05. Note that the coefficient of variation δ_X is calculated as σ_X/μ_X. The characteristic value for strength is then determined using Eq. (12).

Illustrative example

To illustrate the previous approach, consider the common problem of checking the quality of concrete placed in a structure to see if it can fulfill a certain design characteristic strength. It will be most desirable if extensive checks can be carried out quickly and cheaply using a nondestructive test such as the UPV test. Because the sample size is much larger, statistically meaningful conclusions can be drawn. Let us assume that the mean μ_X and standard deviation σ_X of the UPV measurements taken from a particular section are 4.534 and 0.068 km/s, respectively. The coefficient of variation δ_X of the UPV measurements is 1.5%. The characteristic UPV determined from Eq. (11) based on β = 0.05 is 4.422 km/s.

If the acceptable rejection rate for compressive strength is 5% (i.e., α = 0.05), the characteristic strength y_c implied by a characteristic UPV value of 4.422 km/s can be calculated from

Eq. (12) as 35.2 MPa. For illustrative purposes, the model parameters *a* and *b* obtained in this study are used in Eq. (12). The characteristic strength value thus obtained can be compared with the design characteristic strength to determine if the quality of the concrete is acceptable. If the rejection rate is higher, e.g., $\alpha = 0.10$, the value of y_c becomes 40.4 MPa. The computed characteristic strength is more optimistic since the engineer is willing to accept more risk of the concrete not achieving the characteristic strength. On the other hand, if the concrete is more variable, e.g., $\delta_X = 0.02$, the value of y_c becomes 31.3 MPa. This value is more pessimistic because quality control is poorer as reflected by the higher δ_X. More examples on the variation of characteristic strength with risk level and inherent variability of concrete are shown in Table 4. It can be clearly seen that the proposed quality assurance criterion systematically and rationally accounts for the inherent variability of concrete, the reliability of the prediction model, and the risk level the engineer is willing to take in the evaluation of the characteristic strength.

CONCLUSIONS

A laboratory testing program was undertaken to evaluate the probabilistic characteristics of UPV and compressive strength. It was found that the coefficients of variation for UPV and compressive strength remained relatively uniform for the three different grades of concrete under study (35, 55, and 75 MPa). Under well-controlled laboratory conditions, the coefficients of variation for UPV and compressive strength were observed to vary from 1.1 to 1.2% and from 4.2 to 4.9%, respectively. These statistics are likely to be lower-bound values for the inherent variability of concrete. Comparison between the empirical and theoretical normal cumulative distribution functions showed that UPV and compressive strength could be adequately modeled as normal random variables. This observation was further supported by the Kolmogorov-Smirnov goodness-of-fit test.

A probabilistic model to predict compressive strength from UPV was then developed using linear regression analysis. The prediction error in the model was found to be normally distributed with a standard deviation of 10.46 MPa. It is important to note that the model parameters are expected to change with field conditions. Hence, the numerical values of the model parameters reported in this paper should not be used unless there are some field data to substantiate their validity. Using this probabilistic model and the statistics of UPV determined previously, it was demonstrated that a consistent statistical quality assurance criterion using UPV could be developed. The proposed statistical quality assurance criterion was shown to be a natural extension of the characteristic strength concept that is already widely used in practice. In addition, the statistical quality assurance criterion also rationally and systematically took into account the inherent variability of concrete, the reliability of the prediction model, and the risk level the engineer is willing to take in the evaluation of the strength of in-situ concrete and in the investigation of areas of poor consolidation and voids. This proposed method allows the engineer to strike an equitable balance between acceptable risk and cost of remedial measures. Development of a statistical quality assurance criterion using other nondestructive methods is currently underway.

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Table 4—Variation of characteristic strength*

Acceptable risk, α%	Characteristic strength, MPa				
	$\delta_X = 1.0\%$	$\delta_X = 1.5\%$	$\delta_X = 2.0\%$	$\delta_X = 2.5\%$	$\delta_X = 3.0\%$
1	30.0	25.5	20.0	13.9	7.4
5	38.4	35.2	31.3	27.0	22.4
10	42.9	40.4	37.3	34.0	30.4
20	48.3	46.6	44.6	42.4	40.1
30	52.2	51.2	49.9	48.5	47.1

*With risk level and inherent variability of concrete for mean ultrasonic pulse velocity = 4.534 km/s.
Note: δ_X = coefficient of variation of UPV readings.

NOTATION

a,b = slope and intercept of linear regression equation, respectively
D-statistic = maximum absolute difference between empirical and theoretical cumulative distribution function
$D_{5\%}$ = critical *D*-statistic corresponding to 5% level of significance
n = number of cube specimens
r = correlation coefficient from linear regression analysis
SD = standard deviation
UPV = ultrasonic pulse velocity
X = UPV, km/s
x_c = characteristic UPV
x_i = ith UPV reading
y_c = characteristic compressive strength
Y = compressive strength, MPa
α = rejection rate for compressive strength
β = rejection rate for UPV
δ_X = coefficient of variation of X
ε = prediction error from linear regression
μ_X = mean of X
μ_Y = mean of Y
σ_X = standard deviation of X
σ_Y = standard deviation of Y
σ_ε = standard deviation of ε
$\Phi(\bullet)$ = cumulative distribution function of standard normal variety

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